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Mitigating Multi-Path Error by Neural Network

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ABSTRACT

This paper presents a neural network approach for calibration of multi-path delay in GPS-based spacecraft attitude determination. Mitigating the multi-path error represents an important step towards a precise and autonomous LEO spacecraft navigation and control. The INPE and DLR cooperation activities in this direction are overviewed. The algorithm validation by a ground experiment at DLR is specially addressed. Results are presented from digital simulation based on an empirical model using series of spherical harmonics.

INTRODUCTION

This paper presents a new approach for calibration of multi-path delay in GPS based spacecraft attitude determination. The use of GPS receivers as single devices to determine orbit, attitude and time is promising to autonomous navigation and control of a LEO spacecraft. Nevertheless, attitude determination with GPS is not as much accurate in general as it is for orbit and time, mainly due to multi-path interference.

Several GPS based attitude determination algorithms have been developed and tested, most of them using the GPS carrier phase observable[1-4]. More specifically, between-antennas single-difference phase observable from a set of antennas placed over a given satellite surface and linked to a single GPS receiver are processed as attitude observations. In this case, the delay on the carrier phase due to multi-path interference is the most important error source.

Project effort on both hardware and software is a key point to mitigate the multi-path effect over the attitude determination accuracy. Avoiding potentially disturbance sources as much as possible from the antenna's neighborhood and using antenna harness like ground planes and chokers are some of the hardware-related recommendations. On the other side, software improvements have been proposed[5], namely a multi-path calibration procedure based on empirical curve fitting of real data observed under a realistic

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interference scenario. Despite its highly irregular spatial shape[6], multi-path interference delay is a repeatable function of the GPS satellite directions in body frame coordinates only. Therefore, the expected performance of the calibration procedure would depend only of the selected set of calibrating functions. In practice, however, ground test results[5] using a series of surface spherical harmonics, for instance, have shown that beyond the 8th order the number of coefficients of the spherical harmonics series grows too much in face of a minor benefit in terms of accuracy gain.

In the context of the Brazilian space activities, INPE and Paraná Federal University carried out an earlier GPS experiment[7]. The aim of that experiment was to proof the concept of proposed algorithms for position fix[8,9] and attitude determination for spin stabilized satellites[10,11]. With the aid of the DLR facilities, the activities have been now extended to algorithm performance evaluation and improvement, including three-axis stabilized satellite applications. In this paper the main results are overviewed with special emphasis on the antenna calibration problem. Specifically a procedure using neural network[12,13] is proposed to empirically model the multi-path delay.

Neural networks present some particular advantages for curve fitting, due to their ability to approach non-linear functions and also due to the training methods. In fact, neural nets can be trained in real time to compensate for multi-path without necessity of mathematical models. They are also excellent interpolators, and can infer about the multi-path errors with reasonable confidence even in points where no information was provided during the training process.

A proof of concept algorithm is thus developed and tested using a digital simulation scheme based on experimental data. Results of the neural network learning process are shown and compared with an empirical spherical harmonics model. The study represents a contribution towards a more accurate GPS based attitude determination.

PROBLEM FORMULATION

For a given set of unit vectors, namely the reference unit vectors u_k^p in the reference frame and the respective observed unit vectors w_k^p in the spacecraft-body frame at a given sampling time t_k, the attitude matrix is ideally such that:

$$w_k^p = A_k u_k^p, \forall p \in \wp_k, \tag{1}$$

where the index p refers to the PRN that identifies every GPS satellite and $\wp_k$ represents the set of the PRN values of all visible GPS satellites at t_k.

There are several well-known algorithms[14,15] to solve Eq. 1 for A_k. They usually take into account the unavoidable inaccuracies and find the optimal attitude estimate in the sense of the Wahba's problem[16].

Specifically when using GPS for spacecraft attitude determination, the reference unit vectors are the set of unit sight vectors of the p-th GPS satellite for all GPS satellites at sight, as seen from the host spacecraft. This set may be evaluated very accurate from the GPS forecasted ephemeris and the satellite position fix obtained by the GPS receiver. On the other hand, the observed unit vectors may be evaluated from interferometry on the carrier phase GPS observable using a set of antennas linked to a single GPS receiver. The interferometry equation yields:

$$\Delta\overline{\varphi}_{i,k}^{p} = \frac{2\pi}{\lambda} a_i^{\ T} w_k^p - 2\pi K_{i,k}^p , \tag{2}$$

where a_i is the i-th baseline vector between the i-th rover antenna and the master antenna; k denotes the k-th sampling time; λ is the L1 carrier phase wave length; $\Delta\overline{\varphi}_{i,k}^{p}$ is the ideal value of the between-antenna single-difference of carrier phase observable; and $K_{i,k}^p$ represents the integer ambiguity due to the fact that a_i is usually bigger than λ.

The accuracy of a GPS-based attitude sensor device therefore is strongly dependent of the carrier phase observable model. There are two main types of inaccuracy sources on the single difference of carrier phase: deterministic errors and random errors. The deterministic errors could be subdivided in bias effects basically due to offset of the antenna phase center and hardware and antenna cable delay; and electromagnetic effects basically due to multi-path. Nevertheless, without loss of generality, the deterministic error is referred in this paper as being caused by multi-path only.

The observed single-difference $\Delta\varphi_{i,k}^{p}$ is then given by:

$$\Delta\varphi_{i,k}^{p} = \Delta\overline{\varphi}_{i,k}^{p} + \delta\varphi_i(u)\,|_{u=u_k^p} + \varepsilon_{i,k}^p . \tag{3}$$

where $\delta\varphi_i$ represents the multi-path delay and the $\varepsilon_{i,k}^p$ the random error. Typical error magnitudes are millimeters for the random errors and up to few centimeters for the dominant multi-path error.

In order to mitigate the multi-path effect, a ground based calibration procedure may be carried out. A surface spherical harmonics series may be so fitted to real measurements when a priori attitude knowledge is available. In the next section it is presented an alternate approach using neural network.

NEURAL NETWORKS

Artificial neural networks are mathematical structures that simulate some characteristics of the human brain. It is composed by individual processing units called artificial neurons grouped in layers. One or more layers can be present in a single neural net, although in some net architecture, like the dynamic nets for instance, the layers are not easily identified. In feedforward nets each neuron applies an activation function *f* to the sum of the weighted outputs of the neurons in the previous layer. The activation function characterizes the net functionality with respect to its ability to better represent one or other problem. In general, a biased nonlinear differentiable function like the sigmoid or hyperbolic tangent, for instance, is used in the hidden layers while the output layer can have a linear function[12]. One of the most important aspects in neural nets is the way in which the neuron weights are adjusted. Several training methods were developed in recent years, allowing a reasonable net representation of the simulated system with a minimum parameter adjusting. A supervised training method adjusts the neuron weights based on error obtained at the net output and applying some optimization rule. Training consists of an interactive process in which the weights are adjusted by propagating the output error through the network layers. Nonlinear continuous functions can be approximated with a given accuracy by a two-layer neural net with linear function in the output and the sigmoid activation function:

$$f(x) = \frac{1-e^{-x}}{1+e^{-x}} \tag{4}$$

in the hidden layer[17,18]. A feedforward network composed by *l* layers, as shown in Figure 1, can be seen as a mapping function with n_0 input elements and n_l output parameters. If x_i^k is the output of the i^{th} neuron of layer *k*, w_{ij}^k is the weight of the j^{th} input (coming from the j^{th} neuron of the preceding layer) and f^k is the activation function of layer *k*, then:

$$x_i^k = f^k\left(\bar{x}_i^k + b_i^k\right) = f^k\left(\sum_{j=1}^{n_{k-1}} w_{ij}^k\, x_j^{k-1} + b_i^k\right) \tag{5}$$

where b_i^k is the neuron bias that allows the neuron to present a non-null output for a null input.

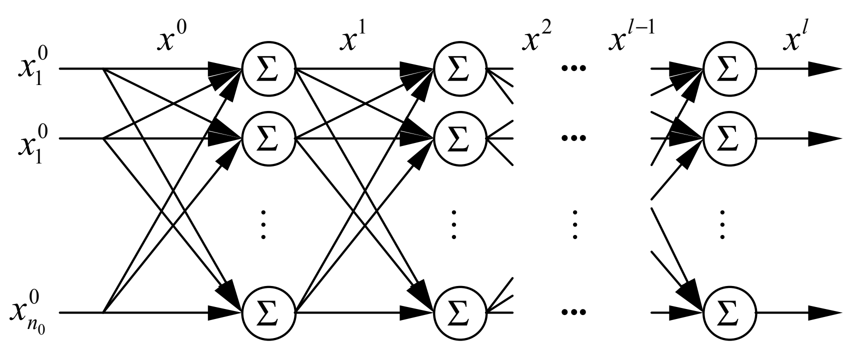

Figure 1   A Feedforward Neural Network

Generally the neuron bias can be obtained together with the weights, by assuming the inclusion of new unit input. In a vector-matrix representation form the preceding equation yields

$$x^k = f^k\left(\bar{x}^k\right) = f^k\left(W^k x^{k-1}\right) \tag{6}$$

where the weight matrix of layer k, W^k, includes the neuron bias:

$$W^k = \begin{bmatrix} w_{11}^k & \cdots & w_{1n_{k-1}}^k & b_1^k \\ w_{21}^k & \cdots & w_{2n_{k-1}}^k & b_2^k \\ \vdots & & \vdots & \vdots \\ w_{n_k 1}^k & \cdots & w_{n_k n_{k-1}}^k & b_{n_k 1}^k \end{bmatrix}. \tag{7}$$

The dimensions of the output vector x^k and the weight matrix W^k are now n_k+1 and $n_k \times n_{k-1}$+1, respectively.

The increasing number of hidden layers normally makes the neural net to better represent the dynamical system and to reduce the output error[19,20], even taking the same number of neurons. Nevertheless, the capacity of generalization, i. e. the ability to interpolate between points where the neural net was not trained is more accentuated on nets with few or even only one hidden layer[21]. The number of neurons in the hidden layers is important for the approximation degree: few neurons tend to decrease the stability and result bad approximation, too much neurons cause oscillation on the output between the trained points[22].

GROUND EXPERIMENTS

Since 1996 experimental works have been developed at INPE in order to validate orbit and attitude determination algorithms based on GPS. The first Brazilian Campaign for attitude determination of spin stabilized satellites using GPS was carried out in 1996 in cooperation with the Federal University of Paraná. Two antennas on a rotating baseline of 60 cm were connected to two separate Ashtech Z-X113 GPS receivers. The integer ambiguity problem was solved and the attitude of the spin axis was successfully determined from double difference of the carrier phase[7]. Afterwards, the experiment was complemented by measuring the pseudo-range from a single antenna with a GEC PLESSEY GPS Builder-2 receiver to test the ORBEST algorithm for position fix[7,8].

Recently the attitude determination algorithm was improved to allow arbitrary high spin rates[11]. The performance of the new algorithm is presently being evaluated more carefully using the results of a new experiment developed in cooperation with the DLR/GSOC (Figure 2). This time two antennas on a 1m baseline could be both connected on a single GPS Beeline receiver allowing the use of single differences.

At the DLR's campaign, several experiment sections have been carried out with the turntable at different spin rates and attitudes with respect to the local vertical and the receiver at different sampling rates. In some of the experiment sections two metal plates heave been introduced at the baseline frame, close enough to the antennas to intentionally generate a stronger multi-path interference scenario (see Fig. 2).

Even if the campaign was primarily intended to test an algorithm for spin axis attitude determination, it was also very convenient to the present study. It would take several hours of measurements for the GPS constellation geometry to change in such way that the multi-path signals coming from the antenna whole field of view could be observed with the turntable at static attitude mode. Actually, due to weather conditions among other practical limitations, that would not be possible at all. Hopefully, the spin motion of the turntable allowed the multi-path on a GPS signal coming from every GPS locked satellite to be observed from all possible azimuth angles in a few minutes only. With this purpose, data from GPS satellites at 8 different elevation angles were stored over a few complete revolution cycles. Based on those data an empirical model was fitted using series of spherical harmonics up to order 15. The model is then used to extrapolate the multi-path at the simulation tests.

Figure 2 GPS Ground Experiment with Multi-path Plates

SIMULATION RESULTS

For the proposed problem, a feedforward neural net with one sigmoid hidden layer and a linear output layer is suitable, although tests will be carried out in future to optimize the net architecture. In the actual problem, the inputs are generated by the GPS receiver, and correspond to the angular position of a given GPS constellation satellite with respect to a body frame. Generally these angles are related to azimuth-elevation coordinates. The neural net is then trained so as to output the multi-path error as a function of the position calculated by the GPS receiver. Once the correction is added to the measurement, at least in theory the multi-path error is then compensated.

In order to validate the process, a simulation arrangement was carried out. In the simulation, a GPS constellation of 24 satellites is considered with orbital parameters as given by Leick[23]. The single difference is computed for a pair of antennas separated by a 1m baseline. The attitude of the baseline is considered known and fixed in the inertial reference frame, each antenna boresight facing the zenith direction. For the sake of simplicity, the integer ambiguity problem is supposed to have been solved by any available algorithm (as in Cohen and Parkinson[24], for instance). The empirical model described in the previous section generated the multi-path effect. Any other delay source is either modeled as a random noise or incorporated as a multi-path-like effect. The standard deviation (1σ) of the random noise at the single difference was approximated by a linear model which reproduces the rms of observed residuals at the experiment, ranging from 0.5mm to 5mm as the GPS elevation angle decreases from 90^o to 0^o.

It was employed a 2 layer feedforward neuron network, with 40 neurons in the first (hidden) layer and 1 output neuron. Activation functions were sigmoid and linear, respectively. Training was carried out with Levenberg-Marquardt algorith[13] and reached a final error of approximately 10^{-5} after 25 iterations. The simulated results are shown in Figure 3, 4 and 5. Figure 3 presents the simulated measured points, in the unit sphere for 24 hours in 60 seconds interval. The simulated multi-path error is shown in Figure 4, obtained with the empirical model. Figure 5 shows the network approximation of the multi-path effect, where it can be seen a great similarity with the preceding values.

The data simulation process was extended by another period of 24 hours in order to evaluate the potential ability of the proposed calibration procedure to predict the multi-path error. These data did not feed the neural network learning process.

The results are shown in Figures 6 and 7. Figure 6 shows the simulated GPS directions on the unit sphere. It presents almost the same pattern as Fig. 3. One should note that due to the attitude stability during the simulation, there are some windows lacking observations. Those windows correspond to the regions with higher disagreement between the calibrating neural network and the empirical model (compare Figures 3, 4 and 5). Nevertheless, as the GPS geometry pattern changes slowly from one day to the next, it is unlikely that many GPS satellites will effectively be in one of those windows in a daily calibration horizon. Therefore, in practice this problem does not seems to affect the algorithm performance.

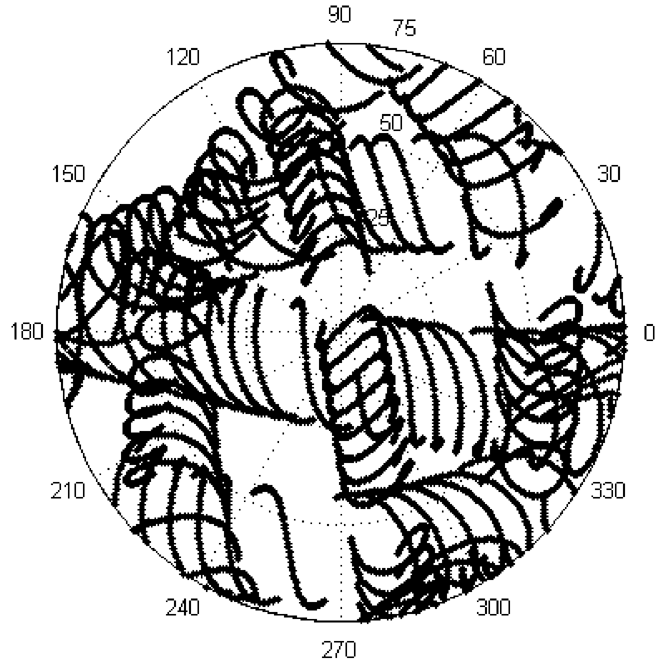

Figure 3 – Simulated Measured Points – GPS Satellite Positions

Figure 7 shows the probability distribution function of the error on the next 24 hours with and without calibration. One can see for instance that there is a probability of 90% that the error after calibration is smaller than about 5mm, against 17mm for the same probability before calibration. Table 1 summarizes the achieved improvement on the accuracy level due to the calibration in the present simulation. The apparent better performance on the whole antenna FOV is in part due to the fact that it does not include the noise effect and also because the 24 hours simulation presented a concentration of data with low elevation, where both multi-path and random errors are higher.

Table 1
RMS OF PHASE ERROR AT SINGLE DIFFERENCE

Domain	Rough Data	After Calibration
The whole antenna FOV	6.2 mm	1.7 mm
Next 24 hours simulation	9.9 mm	3.4 mm

CONCLUSIONS

Results of a neural network learning process to multi-path calibration were shown. The calibration reduces substantially the magnitude of the error on the single difference of carrier phase. The algorithm performance is encouraging and should be considered for on board applications. The study represents a new contribution towards a more accurate GPS based attitude determination. Further analysis and experimental studies are foreseen to fully explore the potential advantages of the neural network approach.

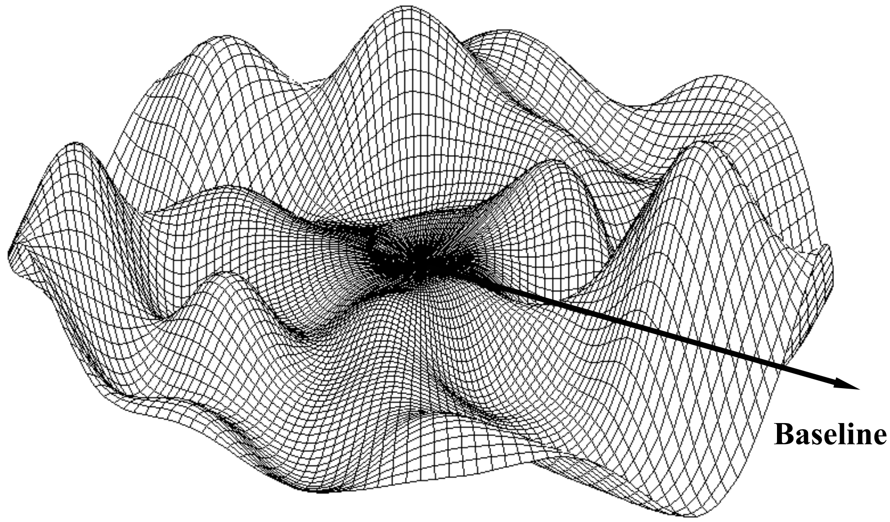

Figure 4 Multi-path Error Modeled by Spherical Harmonic Series Based on Empirical Data

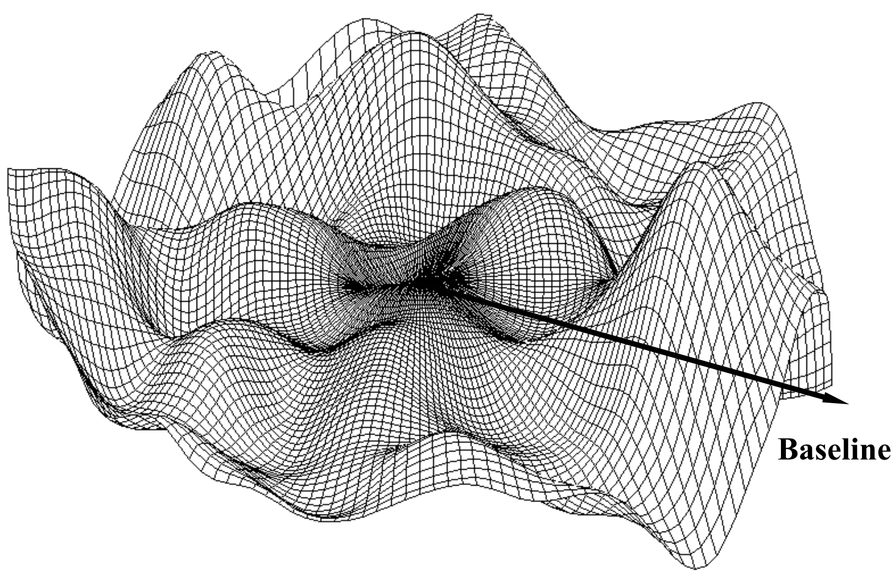

Figure 5 Multi-path Error Estimated by the Feedforward Neural Network

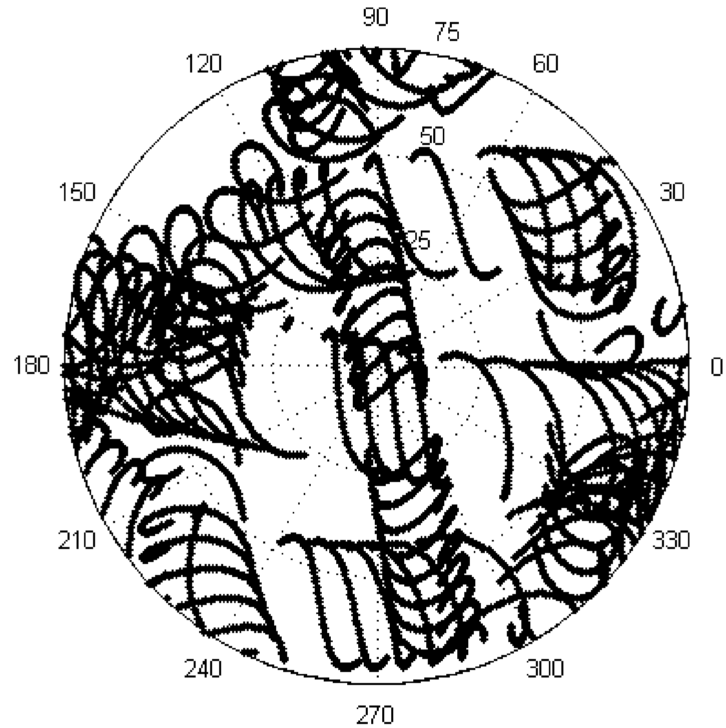

Fig. 6 – Simulated Measured Points for next 24 hours – GPS Satellite Positions

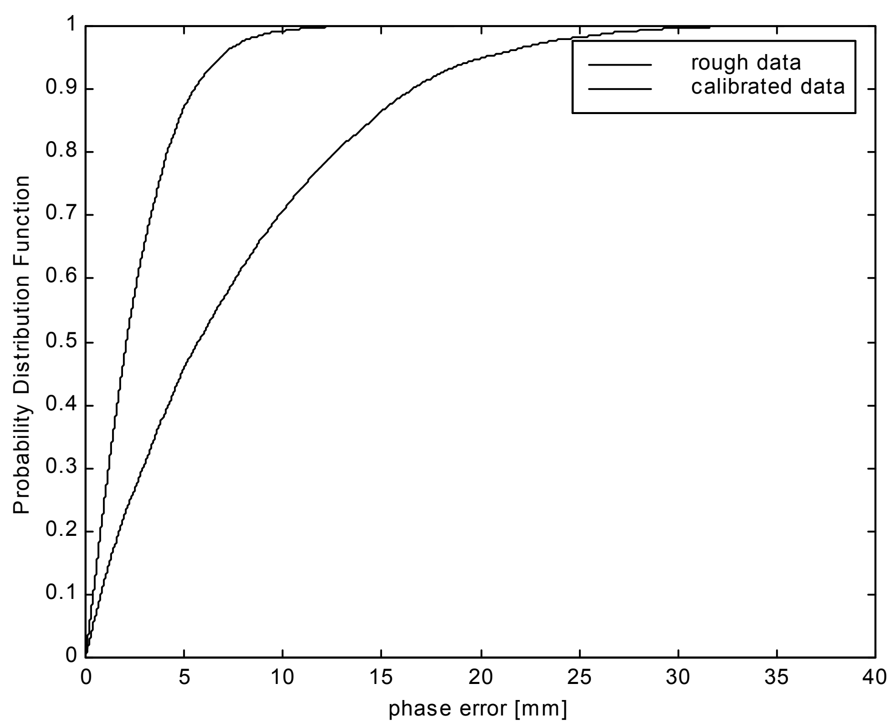

Fig. 7 – Probability Distribution Function of Error at Single Difference Before and After Calibration

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